Viscous Drag Effects on Slender Cones in Low-Density Hypersonic Flow

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Nomenclature

C_D = total zero-lift drag coefficient based on base area and freestream conditions
C_∞ = form of Chapman-Rubesin viscosity coefficient $(\mu_w/\mu_\infty)(T_\infty/T_w)$
D = model base diameter
L = over-all model length
M_∞ = freestream Mach number
p_0 = reservoir stagnation pressure
p_0' = pitot pressure
Re_∞ = Reynolds number based on model length and freestream conditions
T = temperature
u = velocity
$\bar{v}_\infty$ = hypersonic viscous parameter, $M_\infty(C_\infty/Re_\infty)^{1/2}$
θ_c = cone half-vertex angle
μ = dynamic viscosity
ρ = gas density
ψ = nose-to-base radius ratio

Subscripts

E = based on equilibrium flow
N = based on estimates of vibrational nonequilibrium
0 = reservoir stagnation conditions
w = model wall conditions
∞ = freestream conditions

THE large viscous effects on the drag of slender cones in a Mach 10 low-density flow have been presented along with higher Mach number continuum data by Whitfield and Griffith.[1] The purpose of this note is to present new low-density data near Mach 20 for freestream Reynolds numbers between 2500 and 6200/in. (equilibrium values). These zero-lift drag data (Table 1) were obtained in the 100-in.-diam Tunnel *F*,[2] an impulse tunnel of the hotshot type.

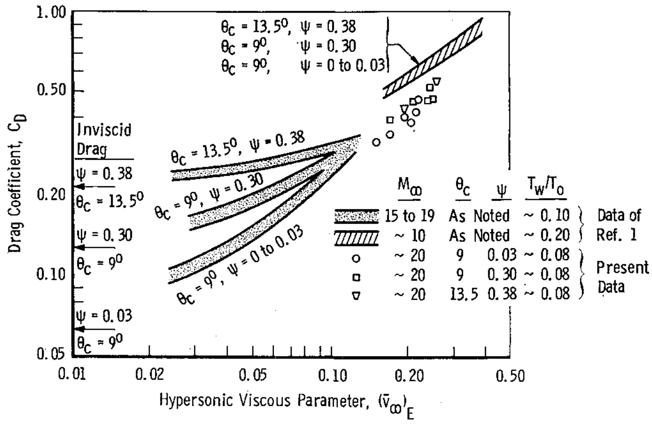

Fig. 1 Viscous drag of slender cones in low-density flow.

The model wall temperature was essentially invariant for the short test times (~50 msec); thus the wall-to-stagnation temperature ratio (T_w/T_0) varied between 0.06 and 0.09, yielding the "cold-wall" condition of practical interest in the hypersonic regime. The present data are presented (Fig. 1) in terms of freestream flow conditions based on equilibrium real nitrogen properties and the method of determining total flow enthalpy discussed by Griffith and Lewis.[3] Estimates of the influence of vibrational relaxation have been made[4] using the relaxation times of Blackman,[5] and the results are shown in Table 2. It is noted from the calculations that the hypersonic viscous parameter $\bar{v}_\infty$ is essentially unchanged due to vibrational nonequilibrium effects. Also, the more recent data of Hurle, Russo, and Hall[6] suggest that estimates of vibrational nonequilibrium based on shock-tube relaxation times, such as Blackman's data,[5] are pessimistic. It should be noted that the ratios of parameters shown in Table 2 are based on equilibrium and nonequilibrium expansions to the same freestream dynamic pressure. The dynamic pressure was obtained, in *all* cases, by the quite accurate (for this flow regime) hypersonic approximation,

$$\tfrac{1}{2}\rho_\infty u_\infty^2 = 0.533p_0'$$

Table 1 Model data

θ_c	ψ	D, in.	L, in.	T_0, °K	p_0, psia	$(Re_\infty/\text{in.})_E$	$M_{\infty E}$	$\bar{v}_{\infty E}$	C_D
9°	0.03	0.97	2.99	4100	7500	5500	20.8	0.152	0.318
9°	0.03	0.97	2.99	3500	4550	4550	21.5	0.172	0.340
9°	0.03	0.97	2.99	4050	3840	3000	21.1	0.209	0.370
9°	0.03	0.97	2.99	4050	3345	2650	20.8	0.218	0.411
9°	0.03	0.97	2.99	4250	3650	2600	20.8	0.220	0.460
9°	0.30	0.97	2.33	4100	7500	5500	20.8	0.172	0.385
9°	0.30	0.97	2.33	3600	3705	3550	21.6	0.221	0.452
9°	0.30	0.97	2.33	4050	3840	3000	21.1	0.237	0.455
9°	0.30	0.97	2.33	4050	3345	2650	20.8	0.247	0.512
9°	0.30	0.97	2.33	4250	3460	2500	20.7	0.252	0.460
13.5°	0.38	1.09	1.65	3850	7990	6250	21.4	0.196	0.423
13.5°	0.38	1.09	1.65	3600	3930	3700	22.0	0.262	0.540

Table 2 Possible vibrational nonequilibrium effects (N_2)

	p_0, psia	T_0, °K	$Re_{\infty E}/Re_{\infty N}$	$M_{\infty E}/M_{\infty N}$	$\bar{v}_{\infty E}/\bar{v}_{\infty N}$
Present data (greatest error)	3600	4500	0.805	0.927	1.035
Ref. 1 (typical)	7700	2500	0.924	0.968	1.006

Received January 28, 1965. This work was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command, U. S. Air Force, under Contract No. AF 40(600)-1000 with ARO, Inc., Operating Contractor, AEDC.
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Presumably, this relationship is independent of the possible nonequilibrium effects. A simultaneous pitot-pressure (p_0') measurement was obtained with each drag measurement; thus the presented drag coefficients are proportional to a ratio of measured quantities.

It is noted from Fig. 1, where comparisons are made with the bounds of previously published data,[1] that the present data are appreciably below the earlier low-density data. This drag reduction is believed due to the relatively colder wall situation for the present data (i.e., the wall-to-total stagnation temperature ratio is less than $\frac{1}{2}$ the value for the earlier data). The present data do not reveal a significant influence of geometry in this low-density regime. This result was also previously noted from the earlier data[1] published by the authors.

References

[1] Whitfield, J. D. and Griffith, B. J. "Hypersonic viscous drag effects on blunt slender cones," AIAA J. **2**, 1714–1722 (1964).

[2] *Test Facilities Handbook* (Arnold Engineering Development Center, Arnold Air Force Station, Tenn., July 1963), Vol. 4, Pt. 6.2, pp. 6.5–6.6.

[3] Griffith, B. J. and Lewis, C. H. "Laminar heat transfer to spherically blunted cones at hypersonic conditions," AIAA J. **2**, 438–444 (1964).

[4] Kinslow, M., Miller, J. T., and Siler, L. G., private communication (July 1964).

[5] Blackman, V., "Vibrational relaxation in oxygen and nitrogen," J. Fluid Mech. **1**, 61–85 (1956).

[6] Hurle, I. R., Russo, A. L., and Hall, J. G. "Spectroscopic studies of vibrational nonequilibrium in supersonic nozzle flow," J. Chem. Phys. **40**, 2076–2089 (1964).

Exact Expressions for Composition of Gas Mixture at Equilibrium

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TEMPERATURE T and density ρ are usually chosen as the two independent variables describing the state of a gas at equilibrium. In practical problems, only the total enthalpy is known, whereas the second independent variable and the composition of the mixture must be determined as a part of the solution. For this reason, the enthalpy and temperature are the most convenient variables for use in homenthalpic flow calculations.

A study of an iterative procedure for the determination of equilibrium composition led to the discovery that closed-form, exact solutions are possible in some simple cases. As an example, consider a five-component mixture of O, N, O_2, N_2, and NO participating in the following three elementary reactions:

$$O_2 \rightleftharpoons 2O \qquad N_2 \rightleftharpoons 2N \qquad NO \rightleftharpoons O + N \tag{1}$$

If n_i are the specific molar concentrations (moles of i per gram of mixture), then the atom conservation equations may be written as

$$n_O + 2n_{O_2} + n_{NO} = C_1 \tag{2}$$

$$n_N + 2n_{N_2} + n_{NO} = C_2 \tag{3}$$

where C_1 and C_2 are constants. The law of mass action gives

$$n_O{}^2/n_{O_2} = (1/\rho)K_1 \tag{4}$$

$$n_N{}^2/n_{N_2} = (1/\rho)K_2 \tag{5}$$

$$n_O n_N/n_{NO} = (1/\rho)K_3 \tag{6}$$

where ρ is the density, and K_1, K_2, and K_3 are equilibrium concentration constants, known functions of temperature only. The enthalpy may be written as

$$h = v_O n_O + v_N n_N + v_{O_2} n_{O_2} + v_{N_2} n_{N_2} + v_{NO} n_{NO} \tag{7}$$

where

$$v_i = (\partial h/\partial n_i)_{T,\, n_j}$$

and where the partial derivative is taken keeping temperature T and concentrations n_j, $j \neq i$, constant. The functions v_i are known functions of temperature only. Considering h and T as given, we have six equations, Eqs. (2–7), for the determination of six unknowns, namely, the density ρ and the five concentrations of n_i. Equations (2, 3, and 7) are linear in the n_i's. If the nonlinear law-of-mass-action equations are not too complicated, one may proceed as follows. Substituting density from Eq. (6) into Eqs. (4) and (5) and introducing as a parameter a new unknown

$$x = n_N/n_O \tag{8}$$

one can use Eqs. (8 and 2–5) to express the unknown concentrations in terms of x. The result is

$$\left.\begin{aligned} n_O &= \frac{2C_1K_1K_3x^2 + K_1K_2(C_1 - C_2)x - 2C_2K_2K_3}{[(2K_3 - K_2)K_1x + K_2(K_1 - 2K_3)]x} \\ n_N &= n_O x \\ n_{O_2} &= \frac{(C_2 - C_1x)K_2K_3}{[(2K_3 - K_2)K_1x + K_2(K_1 - 2K_3)]x} \\ n_{N_2} &= K_1x^2n_{O_2}/K_2 \\ n_{NO} &= K_1xn_{O_2}/K_3 \end{aligned}\right\} \tag{9}$$

Then

$$\rho = K_3n_{NO}/n_On_N$$

$$p = \rho(n_O + n_N + n_{O_2} + n_{N_2} + n_{NO})R_0T$$

To determine the free parameter x, we substitute expressions (9) into the remaining equation, namely, Eq. (7), and obtain a cubic in x:

$$Ax^3 + Bx^2 + Cx + D = 0 \tag{10}$$

where

$$A = C_1K_3(2v_N - v_{N_2})/K_2$$

$$B = \frac{2C_1K_3v_O}{K_2} - h\left(\frac{2K_3}{K_2} - 1\right) + (C_1 - C_2)v_N - C_1v_{NO} + \frac{C_2K_3v_{NO}}{K_2}$$

$$C = h\left(\frac{2K_3}{K_1} - 1\right) - \frac{2C_2K_3v_N}{K_1} + (C_1 - C_2)v_O + C_2v_{NO} - \frac{C_1K_3v_{O_2}}{K_1}$$

$$D = C_2K_3(v_{O_2} - 2v_O)/K_1$$

The roots of the cubic (10) may be expressed in a closed form. Imaginary and negative roots are discarded. However, not every positive root leads to a physically meaningful solution. The mathematical constraints on x are the requirements that all five concentrations n_i be nonnegative. Equating the numerator of n_O in (9) to zero yields two values of x of which only one is positive. This positive value obviously corresponds to a mixture frozen at its low-temperature composition (minimum enthalpy). Similarly, the numerator

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Received January 11, 1965.

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